Information Aggregation and Dissemination in Simulated Markets

By

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Submitted to the Department of Electrical Engineering and Computer Science

In partial fulfillment of the requirements for the degrees of
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Abstract

In this thesis we use an artificial market to perform carefully controlled agent-based experiments which help us gain insight into the process of information aggregation. Within the Rational Expectations framework, we implement two new traders based upon Kernel Regression and Support Vector Regression. We show that, in general, greater sophistication in the learning algorithms used by traders leads to greater profits while facilitating the learning of other agents in the market, thus creating a more efficient market. We also demonstrate cases in which even unsophisticated trading facilitates market efficiency. We lastly show that differing objectives and strategies in the marketplace often make information aggregation more difficult, and make note of the dangers of using complex analytical techniques in problems as poorly understood as finance.

Thesis Supervisor: Professor Tomaso Poggio
Department of Brain and Cognitive Sciences

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Lastly, I would like to dedicate this work to my mother. Mom, thank you for bringing me into this world and always believing in me.

I am so lucky.

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1. Introduction

1.1. Motivation

As our ability to process information continues to grow exponentially, there is a temptation to apply every bit of available technology to capture profit in financial markets. Ph.D's from quantitative disciplines often find that their training is most demanded by hedge funds or investment banks. These same institutions are second only to the military in their contribution towards Artificial Intelligence research, and doubtlessly continually purchase many of the most powerful computers available. These resources are combined to create sophisticated models that attempt to find trading opportunities with any probabilistic trading advantage. With these models in hand, institutional investors employ heavy leverage in an attempt to turn small advantages into sizeable profits, and much like a casino, take as many trading opportunities as possible in order to maximize expected trading profits.

Advanced analysis techniques are not limited to institutional investors, however. A quick search on the Internet reveals "Fast Rocket Science Algorithms using Kalman Filters, Fixed Memory Polynomial Filters + Recursive Fading Memory Polynomial Filters" [i]for use in short-term market prediction. Numerous other vendors abound, all pitching the latest and greatest technology, and all claiming to provide individual investors with tools as advanced as their institutional counterparts.

In light of such advances, academics and practitioners alike continue to debate the incremental value of such technology. Many people assert that the number of intelligent investors equipped with sophisticated techniques searching for profit opportunities dictates that any price in the market must be devoid of arbitrage opportunities and thus efficient.

In the simplest sense however, markets represent a forum for matching supply and demand. All analysis techniques can be thought of as attempts to extract an estimate of the prices resulting from supply/demand imbalances. In this thesis, I hope to not only to gain insight into these advanced algorithms applied to discovery of these prices from a trader's perspective, but also to understand the impact of application of these techniques to a efficiency of the market as a whole.

1.2. Background

Experimental asset markets provide a means to study various properties of financial markets in a controlled manner. In a typical market experiment, human traders participate in a market where synthetic securities are traded. Relevant information regarding the securities, traders' preferences, trading objectives and trading instructions are given. Experimenters design the market experiments by specifying the securities traded and trading objectives. All trading activities are recorded for analysis. The study of market efficiency is one of the most popular topics in experimental markets. In particular, much interest has been focused on studying the efficiency of information aggregation in market with diverse partial information. The main objectives of these experiments is to study how a market mechanism (double auction, for example) effectively aggregates diverse information, and how the traders utilize their private information and learn from market information. The results from these market experiments show that information aggregation can be successful or securities priced efficiently, but only under some conditions. Issues such as (a) the complexity of the securities (in terms of the payoff profiles), (b) the complexity of the markets (in terms of the number of periods) and (c) the complexity of traders' preferences dictates the efficiency of the markets. When information aggregation does occur in the markets, it does not come about instantaneously and usually requires repetition of the same experiment to reach an efficient market. This shows that learning of traders, in particular, learning using market information for the inference of efficient prices, is crucial for the success of information aggregation. Much evidence from the research on experimental markets is based on the assumption that human traders are rational and capable of learning through repetition. However, not much insight has been given regarding how exactly they learn and whether rationality is crucial to an efficient market. For example, consider a market with some 'irrational' agents who ignore private information and only focus on historical information. Technical analysis traders are typical 'irrational' agents in this sense. Although the market experiments are conducted in a controlled environment, it is difficult to acquire information about the actual learning dynamics of the human traders and their trading strategies.

The rational expectations model has been widely used in asset pricing in markets with diverse information: traders possess diverse partial information on the security traded. Under the RE model, rational agents are supposed to condition their beliefs on all available market information including market prices, bid and ask prices, volume and so on. If all traders are rational and observe all market information, the RE model predicts that in equilibrium all traders will be fully informed and the price converges to the full information price (or the RE price). It is commonly accepted that the RE model provides a benchmark aggregate behavior of an asset market with diverse information. The RE model assumes the traders' ability to correctly update their beliefs with market information, and forecast the market equilibrium price. Essentially these traders act like Bayesians in computing their expectations of price. Nevertheless, the RE model suggests neither how exactly the traders go about forming such expectations (the learning of the traders) nor does it provide any theory on the price dynamics (for example, the price path and how quickly price converges). Plott and Sunder (1988) studied markets in which traders trade a single-period, three-state security in a double auction market. Each trader receive diverse, but imperfect information. The market as a whole has complete information. They conducted experiments in which traders may have identical or diverse dividend payoffs. They found support for the RE model when traders have identical preferences, that is, the security pays the same amount of dividend to every trader. Forsythe and Lundholm (1990) studied a similar set of experiments and observed that trading experience (repetition of experiments) and common knowledge of the dividend profiles (in case when traders have diverse dividend payoffs) are crucial for attaining the RE equilibrium. O'Brien and Srivastava (1991) showed that even with uniform dividend payoffs, other complications in the market such as the number of periods, correlations of dividends across different securities, lack of knowledge of information distribution etc. hinder the convergence of prices.

1.3. Our approach

In this thesis, I investigate the learning dynamics and the role of trading strategies in a series of market simulations through the use of computational agents. These agents take the place of human subjects in the experimental markets and interact with each other.

Unlike experimental markets, in simulated markets the characteristics, preferences, trading strategies and population of the traders can be carefully designed and specified. In designing these agents, I focus on their capability to infer the efficient price based on market information. The empirical Bayesian traders are constructed as a basic rational trader under the rational expectations model. These traders act as Bayesians who update their beliefs – the conditional distribution of price – with new market information. By using histograms to approximate the empirical conditional distribution, the empirical Bayesian traders only approximate what the actions of Bayesians. The use of histograms represents just one way of the estimating the empirical conditional distribution. Next I consider building other types of traders using other more advanced estimation methods, namely kernel regression and support vector regression. These agents represent more sophisticated traders who are capable of more accurate estimation given the same market information. Once these traders are created, I create different scenarios to test their robustness over several dimensions: diverse partial information, diversity in the objectives of trader populations, diversity in the learning capabilities of the trader populations and so on. This thesis focuses the effects of complexity of the agents on the efficiency of the markets. The main finding is that complexity of the agents does increase market efficiency in most scenarios. The performance of individual agents, measured in wealth accumulated in a simulation, is also improved with increasing complexity. The results from one experiment show that a less informed but more sophisticated agent performs comparably to a better-informed but less sophisticated agent. In that case, the complexity of an agent effectively compensates for the lack of information. However in a noisy and inefficient market, complexity may deteriorate the performance because agents might simply be estimating the noise.

2. The Simulation

The simulation takes place over many periods. In each period, there is a double auction market in which a single period stock trades. These stocks pay an end of period liquidating dividend that is contingent upon the randomly determined state of the economy. Different traders are given different hints about the state of the economy so that the market as a whole possesses full information while individuals do not. Individual traders trade for a variety of reasons, but primarily to maximize wealth. Such wealth maximizing traders attempt to estimate the value of the security from their private information as well as the information presented in the market place. Upon estimation of the value of the security, they enter trades with positive expected return. As periods pass, traders with learning capabilities are able to learn from their experiences of past periods and revise the way in which they incorporate market information into their formation of an expected price.

2.1. Economy

Every period, the economy can be in one of three randomly determined states, each of which is associated with a different dividend, D={\$1,\$2,\$3}. The true state is in general unknown to the traders, but the underlying distribution function is common knowledge. Specifically, it is uniformly distributed between the three possible values. Before trading commences, the state is determined and hints about the economy in the form of $D \neq D_x$ are distributed to the traders in the market. The hints are distributed at random so that, except for rare chance events, the market aggregately possesses full information, while most individuals do not. Differences in beliefs about the state of the economy lead traders to value the securities differently, creating incentives for the trading in our market. Wealth maximizing agents are expected to buy securities they believe to be undervalued and to sell securities they believe overvalued.

2.2. Market

Securities are traded in a simplified double auction market. Each market consists of a series of rounds. During each round, traders are sequenced randomly and allowed to submit a single order. Orders are of the following four varieties: Bid, Ask, Buy and Sell. Bids and Asks are limit orders reflecting a trader's desire to buy or sell securities at a price specified by the trader. If a Bid/Ask already exists in the market, subsequent Bids/Asks must be more competitive than the existing Bid/Asks, that is, new Bid prices must be higher and new Ask prices must be lower. Buy and Sell, are "market" orders, and represent the willingness of a trader to immediately transact securities at the prevailing Ask or Bid prices. Upon submission of a market order against an existing limit order, the traders exchange shares at the limit price, and the market order cancels the prevailing limit order.

At the beginning of the simulation, each trader is allocated $10 dollars and 5 shares of stock. At the end of each period, wealth accumulated is recorded and stock and cash are reallocated so that the liquidity of the market is independent from period to period. Traders are subject to a strict wealth constraint, and are not allowed to sell-short.

2.3. Traders

2.3.1. Design of Agents

In the design of the agents used in the simulation, we wanted to be able to capture three important attributes that vary widely among traders: objectives and strategies, the information possessed and the methods for updating beliefs.

Objectives and Strategies:

The objectives and strategies found in the marketplace vary greatly from trader to trader and result in hugely different trading behaviors. The first type of trader we consider is a traditional value investor. This type of investor seeks to find under/overvalued securities and buy/sell them before the market has discovered their true value. They might have some fundamental beliefs about the value of the security, but more often than not, their beliefs regarding a security will be influenced by the market. These traders form the basis for our Rational Expectations (RE) traders.

The second objective we consider is that of short term capital appreciation. This type of trader has little interest in discovering the true value of a security, and instead trades on the belief that he can predict the future price of the security. He might believe in trending or mean-regressing markets, and might use any number of technical indicators to determine the future price. For our purposes, we implement a Technical Trader who simply believes that trending markets will continue to trend.

The last objective that we consider is that of liquidity. While it seems strange that a trader might be forced to buy or sell securities, it is in fact quite commonplace. Options dealers, in particular are forced to buy/sell securities to maintain an optimal mix of options and stock. Beyond sophisticated traders such as these, other people with liquidity needs might include companies with a stock purchase plan, or index funds trying to meet specific returns. In all cases, such traders will prefer to buy low and sell high, but find that their liquidity need overwhelms this desire. In our market, we model these Liquidity Traders as agents who buy and sell securities according to some preset random probability.

Private information:

To address the second dimension, namely the effects of information distribution, each trader is able to be equipped with three levels of information: zero information, partial information and full (or inside) information. Zero Information traders possess only publicly available information and must derive all their information from current and past market conditions. Partial Information traders have knowledge of the impossibility of certain states. In our market, if the dividend will be $2, the trader might receive a hint stating that the dividend will not be $1, or that it will not be $3. The last level of information is that of the Insider, or a trader who possesses perfect knowledge of the value of the dividend.

Learning Techniques

To address the question of how traders learn from the market, we implement three different RE traders. Each differs in the methods by which they update their beliefs. The simplest type, Emprical Bayesian Traders (EBT) maintain empirical distributions of moving averages in the market and associate them with previously encountered periods.

Kernel Regression Traders (KRT) are somewhat more complex, extracting more features from the market and using kernel regression to estimate price conditioned upon their information. Support Vector Traders (SVT) represent the most sophisticated traders in the market. These traders perform the arduous task of training complex support vector machines that give even more accurate estimates.

2.3.2. Rational Expectations Traders

Investors are interested in determining the conditional empirical distribution of the future price, $P(S_t \mid I_p, I_m)$, where S_t represents some future price, I_p represents private knowledge and I_m represents the information observed in the market. Given perfect knowledge of $P(S_t \mid I_p, I_m)$, RE traders would maximize their expected utility, rather than expected wealth. To simplify the problem, however, we assume that all traders are risk-neutral, and seek to trade when the resulting expected profit is positive. This simplification allows us to focus on the method by which traders calculate $E(S_t \mid I_p, I_m)$, rather than the entire distribution $P(S_t \mid I_p, I_m)$.

RE traders represent the rational traders found in the market. These traders initially form their expectations of the value of the stock from their private information and revise their expectations as market information becomes available. The updating is done by revising the market information I_m. Ideally market information includes all market variables: trade price, bid and ask prices, volume and etc. Nevertheless, to reduce computational costs in the simulations, I_m is usually reduced to a sub-set of these variables. Once an expectation is formed, the RE traders decide whether to trade by calculating the trading profits of buying and selling in the market $\pi_B = E(S \mid I_{p,} I_m) - P_a$ and $\pi_S = P_b - E(S \mid I_{p,} I_m)$, where P_a and P_b are bid and ask prices respectively. The RE traders attempt to maximize their wealth by using the following trading rules, with a pre-set profit threshold e:

If $\pi_B > e$, Buy at market
Else If $\pi_S > e$, Sell at market
Else If no bid, post a new bid uniformly distributed $\in [P_{b,} \mathrm{E}(S \mid I_p, I_m)]$
Else If no ask, post a new ask uniformly distributed $\in [\mathrm{E}(S \mid I_p, I_m), P_a]$
Else If there is no ask or bid,
flip a coin to decide to buy or sell,
Randomly price such that expected profit uniformly distributed $\in [0,1]$.

2.3.3. Liquidity traders

Liquidity traders trade purely for liquidity reasons. Each round, they have some preset probability to trade. In rounds in which they are chosen to trade, they first attempt to trade against existing limit orders, flipping a fair coin to decide in the presence of both a Bid and Ask. If no limit orders exist, the trader flips a fair coin to decide between posting a Bid or Ask, and subsequently posts a random limit price centered near the last transaction price. In the event that no transactions exist, the trader will post a random limit price uniformly around the conditional expectation $\mathrm{E}[S \mid I_p]$.

2.3.4. Technical traders

Technical traders are simply traders that believe today's returns will equal yesterday's. Specifically, they calculate the return on the previous two transactions, and form their expected price on the assumption that the previous return will hold for the next transaction. Given this expected price, they submit orders in the same manner as RE traders.

3. Agent Learning in the Artificial Market

This section presents on the learning of the agents, and in particular the computing of conditional expectations of the efficient price given some market information. Here I consider three different ways of calculating the conditional expectation by the use of histograms (Empirical Bayesian Trader), kernel regression (Kernel Regression Trader) and support vector regression (Support Vector Trader). The two regression traders represent more complex and sophisticated traders.

3.1. Empirical Bayesian Trader (EBT)

The Empirical Bayesian trader is borrowed from Chan et al. When implemented, the trader estimates the conditional density functions by constructing a histogram associated with every state. Each histogram represents the distribution of moving averages encountered in each dividend state. During the experiment, the moving average prices are recorded. Once the true state is revealed, the corresponding histogram is updated with the new series of moving average prices. By participating in more experiments, the empirical Bayesian traders attain more accurate estimates of the conditional probability. Intuitively, the empirical Bayesian traders learn the state by associating moving averages with the realized state.

The only feature extracted from the market which is used in formation of the $E[P_f]$ is the sequence of moving average prices. As such, the only parameter to choose is the number of trades used in the calculation of the moving average price. Because we wish to provide benchmarks for our experiments comparable to those in Chan et al, we use 10 transactions in the calculation of the moving average price.

The creation of the histograms used to estimate the empirical price distribution represents a somewhat greater obstacle. These histograms allow us to generalize moving average price data, an intrinsically continuous function, into a few discrete bins. The risk is in under- or over-generalization. A greater number of bins will require a greater number of data points before meaningful changes occur appear in the histograms, while a lesser number might result in characterization loss which could lead to different market conditions appear more alike than desired. We again follow preset convention, choosing 10 bins for every state, or a total of 30 bins.

Initialization of the height of these bins represents another challenge. Initializing these heights to too low of a number will make it too sensitive to early trading rounds. For example, if we initialize the bin heights to 0, we will find that the noise in the first round dominates market and even private information in the next round. If however we initialize the bin heights to too great a number, we will find that the relative changes in the histograms brought about by market participation will be too small, and the market will evolve very slowly. In all the experiments described, we initially set the bin height to 50.

3.2. Kernel Regression Trader (KRT)

The Kernel Regression Traders base their estimates on the non-parametric estimation technique for which they are named. These traders calculate an expectation for the market dividend by taking an average of previous dividends, weighted by the degree of similarity between previous and current market conditions. More complicated than Empirical Bayesian traders, these traders remember price, moving average, momentum, time, and private information with each market transaction.

Kernel regression, also known as Nadaraya-Watson estimator, is a nonparametric estimation technique. A kernel regression model has the form

$$E[Y \mid X = x] = \frac{\sum_{n=1}^{N} K_\sigma(x - X_n)Y_n}{\sum_{n=1}^{N} K_\sigma(x - X_n)}$$

$(X_1, Y_1), (X_2, Y_2)$ and (X_N, Y_N) is the set of examples, and K is the kernel that is a continuous, bounded and symmetric real function which integrates to the value of one. In the experiments, Gaussian functions $K_\sigma(u) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{u^2}{2\sigma^2}}$ of a given variance σ were used as kernels. The variance, known as the "bandwidth" of the Gaussian kernel, is an important parameter in kernel estimation. In fact, it plays a similar role as the number of parameters in parametric methods. Large values of σ will result in poor modeling of the data (high bias), but small values will likely result in over-fitting the data (high variance). Alternatively, the bandwidth can be considered as a smoothing parameter. Large values will flatten out the estimated function while small values will cause the function to be unstable. The choice of the bandwidth dictates the balance between bias and variance in kernel regression. Unfortunately, there is no general theory for choosing the bandwidth, which is highly dependent on the data. Nevertheless, kernel regression is a very appealing technique because there is no "training" involved because the regression function is written explicitly as a function of the data points.

Besides having at their disposal a more sophisticated technique for calculating conditional expectations, the KRT also extracts more features from the market. For every transaction that it encounters, it remembers: Average Price, Time of transaction, second moment of transaction prices and the last two transaction prices. When recorded in the KRTs history of transactions, all values were normalized to be roughly ± 1, and σ was chosen to be 0.2.

3.3. Support Vector Trader (SVT)

Support Vector Regression is obtained by generalizing the Support Vector classification technique to regression estimation. The main idea of support vector classification is to map the training data into a higher dimensional feature space via some kernel function and construct a hyperplane separating the examples with maximum margin. In support vector regression, an analogue of the margin is constructed by using a loss function known as ε-loss function:

$$| y - f(x) |_{\varepsilon} = \max\{0, | y - f(x) | - \varepsilon\}.$$

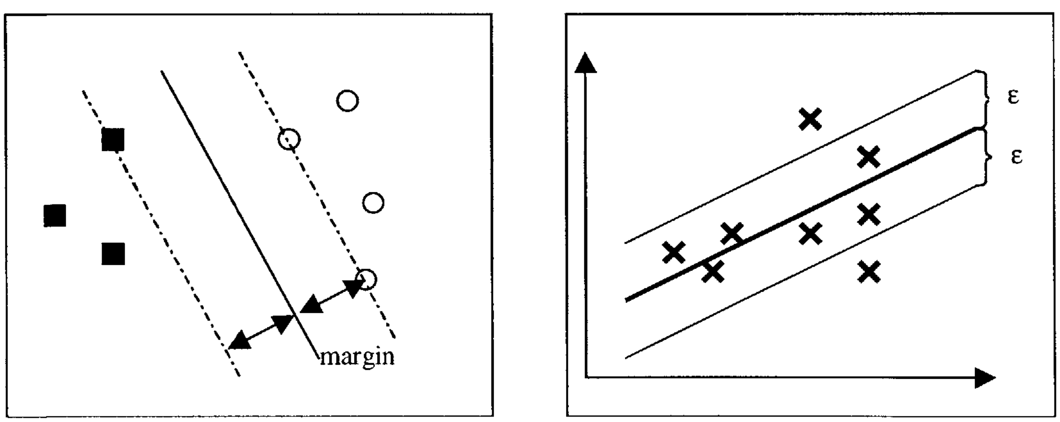

Figure 1 (a) In support vector classification, the idea is to find an optimal hyperplane that maximize the margin, which is the shortest Euclidean distance between the hyperplane and the examples. (b) In support vector regression, a constant ε is specified as the desirable accuracy, and a tube of radius ε is fit to the data.

For example, to estimate a linear regression function

$f(x) = \mathbf{w} \cdot \mathbf{x} + b$

with precision ε, the corresponding optimization problem becomes

$$\frac{1}{2}\|\mathbf{w}\|^2 + C\sum_i | y_i - f(\mathbf{x}_i) |_\varepsilon .$$

Essentially the algorithm penalize, according to the variable C, any examples that deviate from the regression function more than ε. The precision variable ε controls the complexity of the regression function. Generalization to nonlinear regression estimation is achieved by rewriting the regression function in terms of a kernel function.

SVM Regression traders remember exactly the same information as Kernel Regression traders and, once trained, behave in the same manner. An important difference between the two trader types lies in the geometric interpretation of the support vectors produced from training data. SVM Regression essentially calculates the tube (defined by ε) within which all support vectors are known to lie. If, however, a point for which we require an estimate is located in a part of the input space that was poorly populated during the training phase, our estimate of the location of the tube could be very poor. The resulting estimate could be completely unrealistic. Great care must be taken to have a training space that is properly populated. For this reason, we choose to disallow any SVM trader from participating in markets until a sufficient data set has been established.

The SVT was tuned in the environment described in Experiment 2, consisting of a single partially informed SVT competing with 19 EBTs. The parameters describing were the kernel function, C and ε. After some experimenting, we chose a 2^{nd} order polynomial kernel function, a cost of C=1.2 and precision ε=.1.

4. Experiments

The aim of this thesis is to gain insight into the aggregation of prices in the securities markets. To attempt to make this problem more tractable, we make broad and sweeping assumptions including: risk neutrality, easily valued single-period securities, homogenous preferences and a transaction size restriction of a single share. Even with these restrictions, the number of possible experiments is staggering. Simply by altering the parameters of the RE traders, we could create a virtually unlimited number of traders with different learning characteristics. In order to conduct meaningful experiments, we first begin by reconstructing an experiment from the literature that is well understood. Once this is done, we attempt to conduct interesting experiments that vary from this benchmark in as few dimensions as possible, so that we are better able to make judgements about the results of our experiments. With this theme in mind, the first two experiments following the benchmark will involve only binary replacement of individual traders. These two experiments aim are designed to test the learning and information aggregation properties of the Support Vector and Kernel Regression Traders. The last experiment, exploring the difficulty of information aggregation in a noisy market with irrational traders, necessitates slightly more complex replacement, but again, the only difference is in the composition of the population.

Each simulation described lasts 100 periods, and is executed 100 times, with summary statistics being saved at each step.

Upon completion of each experiment, we are interested in understanding how quickly, and with what degree the market is becoming efficient. We are also interested in the relative abilities of certain traders to accumulate wealth. To these ends we compute two statistics, which are calculated for each of the 60 periods, and averaged across the 100 simulations.

Price Deviation: We choose to define price deviation as the average of the deviation of the last two trade prices in the market from the value of the security being traded, or

$$\text{PriceDeviation} = \frac{|P_f - P_T| + |P_f - P_{T-1}|}{2}$$

Relative Wealth Performance: Because the wealth in the market from period to period is random, it is hard to compare absolute wealth measures across experiments. Instead, we choose to compare the accumulated wealth of 2 groups of traders, via the following formula:

$$\text{WealthPerformance}(Group1, Group2) = \frac{\frac{1}{n_{Group1}} \sum^{Group1} wealth_i - \frac{1}{n_{Group2}} \sum^{Group2} wealth_j}{\frac{1}{n_{Group2}} \sum^{Group2} wealth_j}$$

4.1. Benchmark Experiment

4.1.1. Design of Experiment

The objective of the first experiment is the creation of a market to which we will benchmark all other experiments while replicating experiments done in experimental literature. [Chan et al, 1998] In this experiment we choose a trader population which is composed of 20 Empirical Bayesian Traders, each endowed with diverse partial information. In aggregate, the market has full information of the value of the stock, but no individual possesses this information. As we do not equip the EBTs with any initial ability to extract information from the market (by setting all bin heights to be equal) we initially expect very weak signs of convergence in the market. As trading commences, and these bins begin to differentiate themselves, traders will begin to form better estimations of the correct value of the stock. In turn, they will trade more intelligently and their actions will be more easily interpreted by subsequent traders. We expect this process to be positively reinforcing, that is average efficiency in the market should be strictly increasing once a the EBTs have sufficiently trained.

4.1.2. Results

After running this experiment, we look to a graph of average price deviation (Figure2) for evidence of price convergence. Though quite a bit variance exists in the period-to-period average price deviation, the average price deviation is, in general decreasing as time passes.

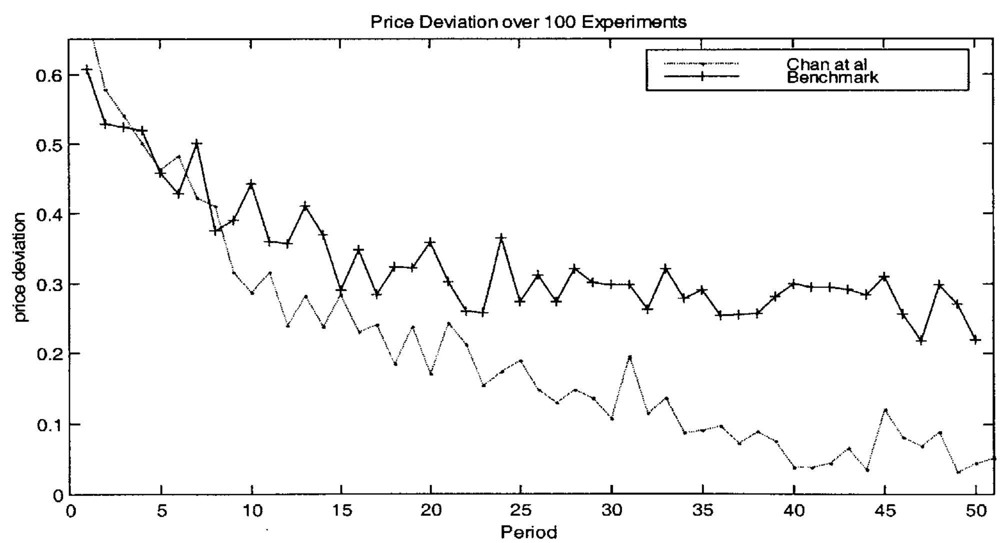

Figure 2 Comparing our benchmark experiment to those by done by Chan at al.

There are, however, two obvious differences between our experiment and the equivalent performed by Chan at al. First, initial price deviations in our experiments are somewhat lower, and second, price deviations show less rapid decay over time. The first difference can be explained by a change of metric: instead of averaging the absolute price deviation of the last 20 trades in a period, we choose to average the last 2. The second difference is accounted for by minimum expected profit constraint placed on our REE traders. This constraint forbids REE traders from entering into trades unless the expected profit from the trade is above some threshold, in our case $.03. This change reduces the number of trades in the market, subsequently slows the rate with which the EBTs empirical histograms are updated. This change also prevents them from attempting to extract the very last cents from a trading opportunity, in the event that a market has converged, adding at least $0.03 to the price deviation statistic in convergent markets.

4.2. Testing SVT and KRT in the benchmark environment

4.2.1. Design of Experiment

In this group of experiments, we want to evaluate the impact and performance of each REE trader by replacing a single EBT trader with a different trader.

We first begin with the mix of traders found in the benchmark experiment and replace 1 EBT with an SVT or KRT. We do not allow the SVT to begin trading until the 24^{th} round. Prior to its first trade in the 24^{th} round, the SVT will train once on the data accumulated through the first 23 rounds. Because of the intense computational requirements of Support Vector Machine training, the SVT does not sequentially retrain. The KRT is allowed to trade from the beginning, and retrains each round. To better compare the performance of these traders relative to each other, we also conduct an experiment in which both traders are inserted into the Benchmark trader mix, in place of 2 EBTs.

The SVT and KRT both have substantial advantages over EBTs in the information extracted from the markets. Besides considering average price information, they also keep second order statistics, time information (in the form of arrival number of the trades within a period) and the last two trade prices. With these informational advantages, as well as the sophistication of their algorithms, we expect both traders to outperform the EBTs in the market.

When we insert both traders into the same market, we expect an interesting wealth accumulation contest. We expect that both the SVT and KRT will continue to accumulate more wealth than the EBTs, but we are more interested in each trader's effect on the other. In particular, the SVM has the advantage of being able to witness the market with the presence of the KRT, while the KRT will have no knowledge of its presence until it enters in the 24th period. The KRT however, will have the benefit of sequential retraining.

4.2.2. Results

The relative wealth performance of the KRT is initially negative, but becomes strictly positive after five rounds, (Figure 3a) This initial disadvantage can be explained by the lack of past price information in the for the KRTs to base their decisions. The KRT begins without any initialized data points, (equivalent to the EBT initial bin height) and is thus more influenced by initial market conditions.

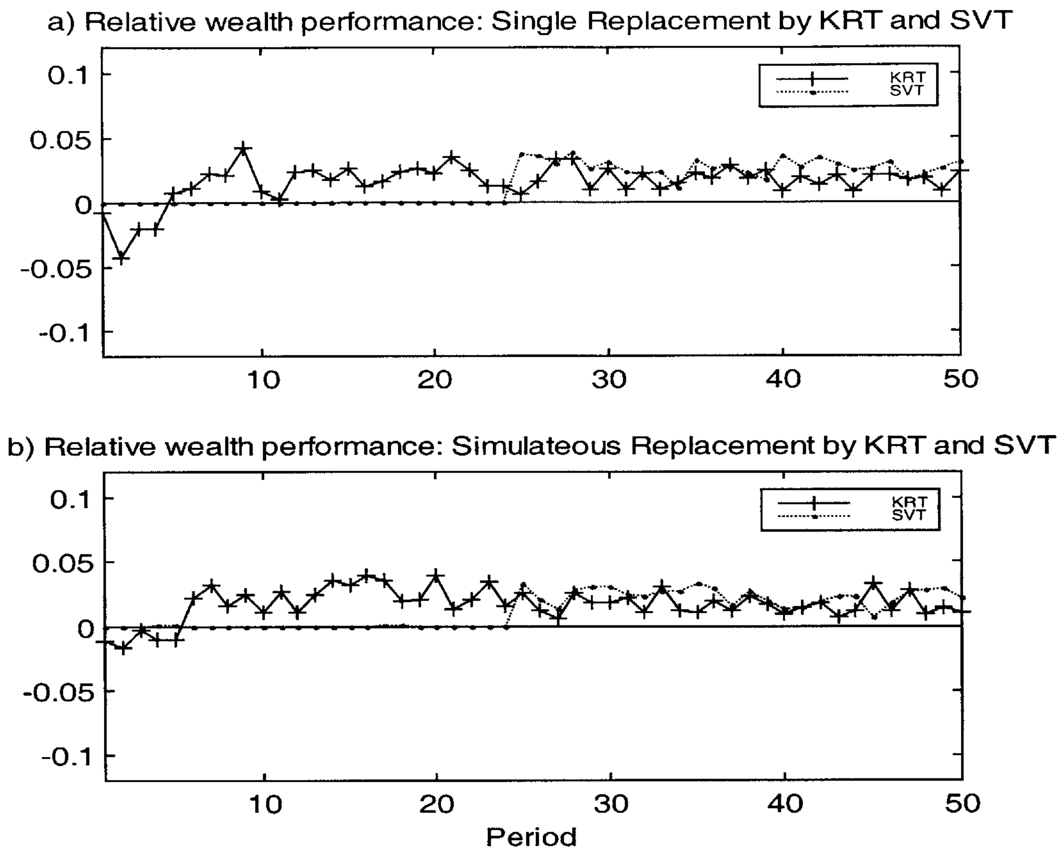

Figure 3: Relative Wealth Performance of individual KRTs and SVTs replacing EBT traders from the benchmark experiment. a) shows the wealth performance when the SVT and KRT are replaced singly. b) shows the wealth performance of each type when they simultaneously replace EBTs.

In the process of extracting wealth from the market, the KRT also disseminates its beliefs through its trading. It is notable that the Average Price Deviation found in the market (Figure 4a) containing the KRT is uniformly below that of the benchmark market, until period 5, the period corresponding to the first signs of positive wealth performance by the KRT. While the relative wealth performance (in the rounds in which it trades) of the SVT is consistently higher than that of the KRT, (Figure 3a) it is not quite fair to immediately state that the SVT is superior in extracting market information. Because we force the SVT to stay out of the market for the initial 24 rounds the EBTs in the market have no ability to adjust to SVT. Thus the SVT allows the EBTs to persist in a semi-efficient state while learning how to extract price information from their actions. The SVT then enters the market and takes advantage of the lack of efficiency in the market. This contrasts greatly with the behavior of the market with a KRT in which the KRT continually takes advantage of the market's inefficiency, but, through its own trading helps teach the market how to become more efficient, thereby degrading its advantage.

Adding both the SVT and KRT to the same otherwise homogenous market provided a better benchmark for comparing the two algorithms. We show that in the rounds in which it trades, the SVT maintains a wealth performance advantage over the KRT (Figure 3b). While it is expected (for reasons explained above) that this result would occur just after the introduction of the SVT, it is interesting that the retraining ability of the KRT does not allow it to diminish this difference. In the absence of this occurrence, we can infer that either 1) the market is predictable, and the SVM generalization is so well constructed that no amount of training will allow the KRT to overtake the SVT or 2) the KRT, although able to learn rapidly, has already learned too much from the market to be able to adapt to the presence of the SVT.

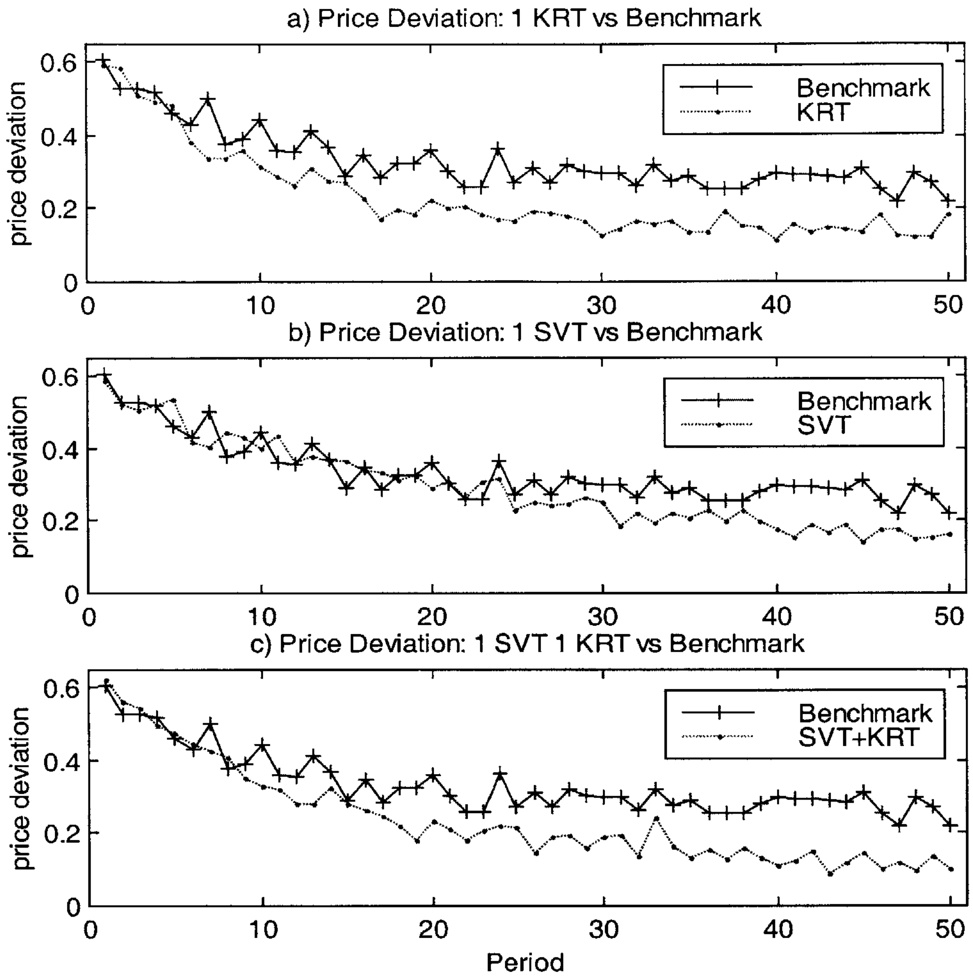

Figure 4: Average Price Deviations of markets which deviate from the Benchmark experiment by substitution of other PI RE traders. a) substitution of 1 EBT by 1 KRT. b) substitution of 1 EBT by 1 SVT and c) substitution of 2 EBT traders by a 1 KRT and 1 EBT.

Having discovered that both the KRT and SVM contribute to efficiency in the price discovery function, its unsurprising that the combination of both traders provides a market which shows faster convergence than markets containing only one of the traders. (Figure 4c)

4.3. Uniformed RE Traders

4.3.1. Design of Exeriment

In this experiment, we evaluate the ability of uninformed REE traders competing with partially informed EBTs. This can be thought of as an experiment to determine the value of sophistication of a trader versus the value of information. Again, we use the benchmark experiment as a beginning point, replacing a single partially informed EBT with either a ZI SVT or a ZI KRT. To get an interaction free picture of the value of partial information, we also conduct an experiment in which we introduce a ZI EBT in place of a PI EBT.

We expect that the ZI EBT will be at a severe disadvantage when competing with better informed EBTs. As before, we expect the sophistication of the SVT and KRT to account for improved wealth accumulation over a similarly informed EBT, but the degree of this advantage is difficult to predict.

4.3.2. Results

Wealth figures (Figure 5b) from the experiment show that the EBT trading with zero information suffers a substantial disadvantage from the outset of the experiment. As the market becomes more efficient however, this disadvantage is mitigated by the presence of more efficient prices in the market, and the losses of the EBT subsequently decline. Comparing the price deviations of this experiment (Figure 5a) to those from our benchmark experiment, we note a sizeable increase in the rate at which the market is becoming convergent. It is not intuitive that replacing an agent with another agent, identical in all respects except that it has less information, and consequently makes poorer decisions, will bring about increase in efficiency. The answer lies in the nature of the learning technique of the EBTs, which continually build upon their existing histograms. By trading poorly, the ZI EBT is creating more trading opportunities for better-informed EBTs in the market. With the increased trading in the market, EBTs construct their histograms faster, and consequently trade more efficiently. To confirm this result we conducted a separate experiment in which we replaced the uninformed EBT with a Liquidity Trader. Trading randomly, but losing even more in the average, the LT's trading also resulted in faster convergence compared with the Benchmark market.

When we replace the uninformed EBT with a KRT, we find that the KRT's relative wealth performance (Figure 5b) generally oscillates around zero, implying that it is not deterministically gaining or losing to the better-informed EBT traders, but rather holding its own. This trader has effectively overcome its informational disadvantage with sophistication in its learning process. As in the case with the ZI EBT and LT, we note a marked improvement in the speed of market convergence, even over that found with other ZI traders.

This result contrasts starkly with that of the SVT. Upon entering the market, the SVT immediately extracts profit from the market in every round in which it trades. Such a result is a clear indication of the ability of the Support Vector Regression to generalize this problem. While the price deviations brought about by the SVT are somewhat higher than that of the those found in the market with the KRT, the decreased number of trading periods for the SVT make it unfair to state that it is less effective in bringing about price convergence. The most interesting result of the introduction of the ZI SVT to this market was the discovery that the wealth performance is actually greater than that achieved in similar markets in which the SVT had partial information.

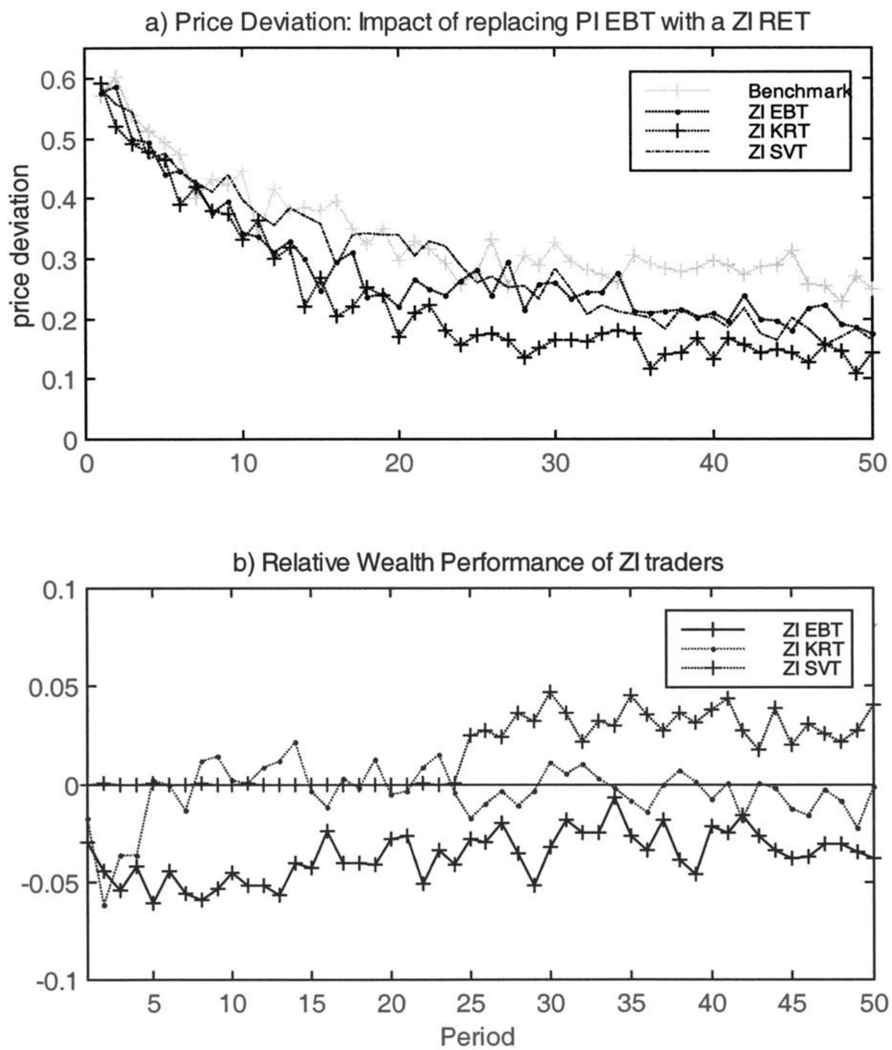

Figure 5: Results from ZI experiments a) Price Deviation of markets containing a single zero information trader b) Relative Wealth Performance of non-EBT traders.

4.3.3. Discussion

The final result of this experiment highlights some of the dangers of using complicated analysis tools in applications as poorly understood as financial markets.

Under the RE model, agents are expected to condition their expectations upon both private and market information. For this reason, it is intuitive that giving an agent incremental additional correct information should never adversely effect the agent. In this case however, much of the information contained in the additional regressors (ie private information) has already been extracted by the SVM from the market information. Adding private information thus becomes a tradeoff between supplying incremental private information not already extracted, and creating a more complicated input space from which the SVM is to generalize. In this case, the extra regressors result in an inferior solution. Given the unknown shape and dimension of the geometry of this problem, and the subsequent mapping of this problem into an equally unknown space, it is somewhat unsurprising that such a major feature of the geometry of this problem has previously eluded us.

4.4. Noise and irrationality

4.4.1. Design of Experiment

In all previous experiments, trader populations have been composed of RE traders that begin with a belief about the dividend and constantly update that belief based on the information presented in the market. Thus every trade reflects information about the (rational) beliefs of two parties in the market. This experiment is meant to be a better reflection of markets seen in the real world, in which it is difficult to make assumptions about the rationality or objectives of traders. We begin with the benchmark mix of traders, and alter the population by gradually replacing EBTs with equal numbers of technical and liquidity traders. By increasing the amount of noise and irrationality, while decreasing the number of REE traders we expect to create markets that converge much more slowly. At some point, we expect the noise provided by the non-RE traders to mask the information transmitted by the RE traders, at which point the market will show little sign of evolving into a convergent market. Upon discovery of this threshold, we will alter the threshold experiment by replacing a single EBT trader with either a SVT or KRT.

4.4.2. Results

We first show that the process of price convergence of the benchmark experiment is slowed greatly by increasing replacement of EBTs with Liquidity and Technical Traders. (Figure 6a) When EBTs become the minority in the population, (8 EBTs, 6 Technical, 6 Liquidity,) we fail to see any strong signs of price convergence after the first 20 rounds. We choose this market as the threshold market into which we will test our KRT and SVT.

Upon replacement of an EBT by either a SVT or KRT, we compare the Price Deviation of these markets. With the addition of the SVT, there is no sign additional convergence (Figure 6c), and minimal signs of additional convergence in the case of the KRT (Figure 6b.) It is somewhat unclear that the KRT is actually contributing to efficiency of the market until we examine the relative wealth performance of the KRT and SVT (Figure 6d). While the Relative Wealth Performance of the SVT oscillates from negative to positive, making it unclear as to its performance, the KRT's relative wealth performance is strictly positive for periods greater than 7. This result implies that the KRT is actually being more effective in estimating the true price in the market, and corroborates the observation that the market possessing the KRT shows slight signs of additional convergence.

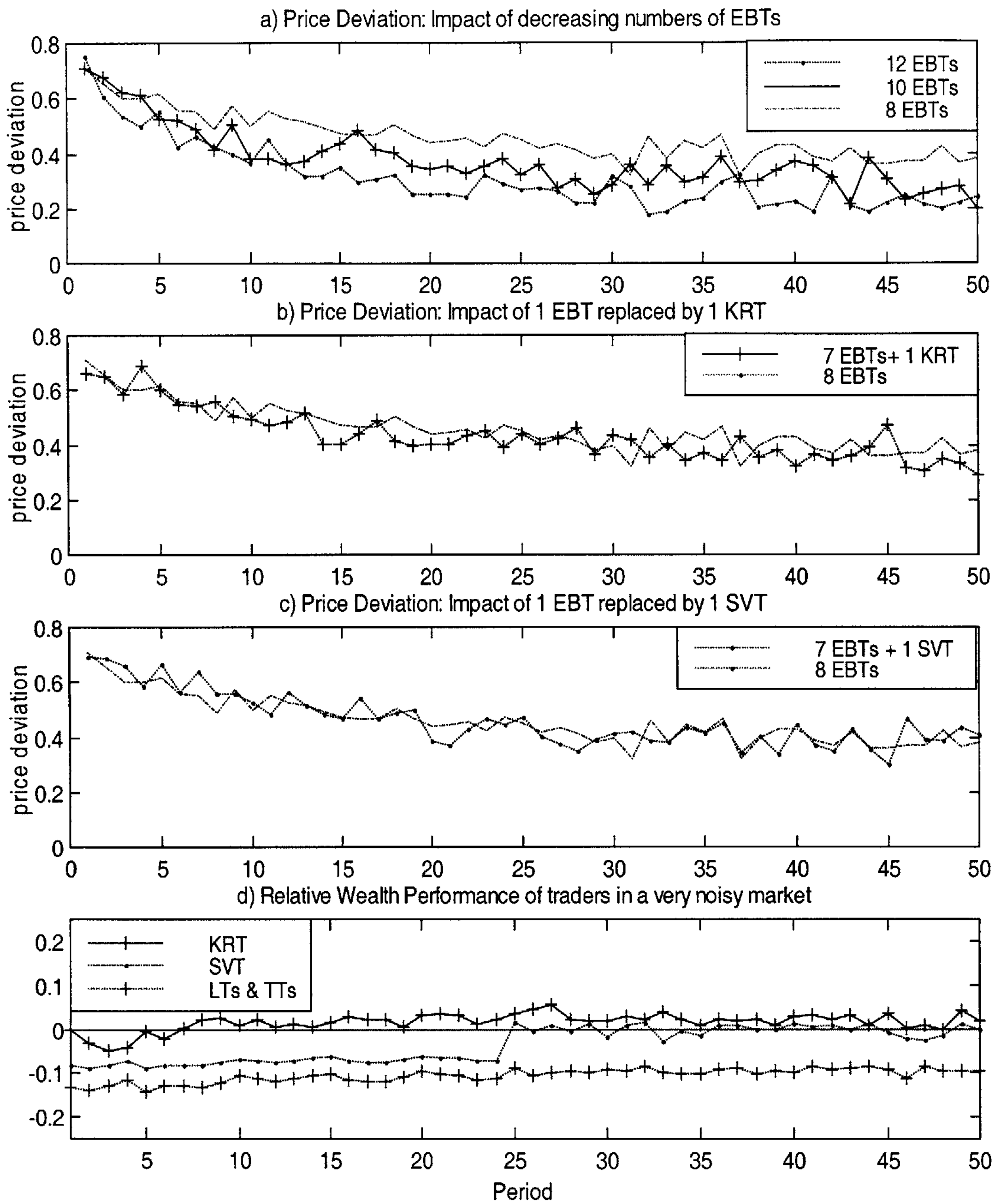

Figure 6: a) Effect of LT and TT on Price Dev. b) and c) Effect of KRT and SVT on a market with 6LTs and 6 TTs c) Wealth of non-EBT traders compared to other EBT traders.

4.5. Notes on Computation

This last set of experiments in particular warrants some discussion of technical issues encountered. The first issue has to do with the sheer complexity of attempting to solve this problem in the face of the amount of noise found in this experiment set. Markets composed entirely of simple traders (LT, EBT and TT) required no more than an average of 4.5 seconds per simulation. Experiments in markets without abundant noise took roughly 10 and 80 seconds for the KRT and SVT respectively. The final experiment of this thesis, consisting of a SVT in the noisiest environment which still exhibited minute signs of convergence, took an average of 12 minutes, with great variation in run times. Certainly some of the increase in time comes about from the fact that the noisiest market exhibited roughly twice as many trades as exhibited in more idealized markets. Even assuming that training an SVM is an $O(N^2)$ problem, doubling of the training set size should only account for a 4x increase in run time, leaving a 2.5x increase in training time that can be attributed to the complexity of the problem.

For reference, all simulations were executed on a 450 Mhz Pentium II computer, with 384 Megabytes of RAM, running Linux 5.2. All market infrastructure was written in Java and compiled with JDK 1.1.6. The only non-Java code in our implementation was a library built by Edgar Osuna used for the training of Support Vector Machines and subsequent estimations. The interface to this library is written in C, but the library makes extensive calls to a FORTRAN QP solver, Minos 5.4. Both Minos and Osuna's SVM interface were compiled with the GNU EGCS compiler for libc.

5. Conclusions

In this thesis we use an artificial market to perform carefully controlled agent-based experiments which help us gain insight into the process of information aggregation. Within the Rational Expectations framework, we implement two new traders based upon Kernel Regression and Support Vector Regression. We show that, in general, greater sophistication in the learning algorithms used by traders leads to greater profits while facilitating the learning of other agents in the market, thus creating a more efficient market. We also demonstrate cases in which even unsophisticated trading facilitates market efficiency. We lastly show that differing objectives and strategies in the marketplace often make information aggregation more difficult, and make note of the dangers of using complex analytical techniques in problems as poorly understood as finance.

There are a few shortcomings of this framework that are made glaringly apparent by this research. First, the lack of risk preference specification of the agents is an apparent limitation of the modeling. We have shown that the simple act of trading, whether rational or not, is vital to the aggregation of information. Yet throughout the experiment we have used the assumption of risk neutrality and budget constraints imposed on the agents to make the problem tractable despite the fact that risk preferences play a huge role in the formation of trading decisions. Precisely for the lack of preference specification, the issue of transaction sizes cannot be addressed. Second the assumption that traders behave like perfectly rational Bayesians may not be a valid claim in reality. Simply put, traders under the framework of the Rational Expectations model may not exist in real markets. A natural extension to these simulations is the inclusion of agents with bounded rationality. Third, the simple setting of the market (single period, single security) may not be able to give insights to more realistic and complicated market situations. For instance, agents in thesis focus on maximizing wealth in each independent runs, instead of the accumulated wealth over multiple runs. The intertemporal characteristics of the market are largely missing.

To its credit, the the real strength of simulated markets is its ability to harmlessly harmlessly model certain events. A future project with real market implications might, for example, be modelling the effects of the increasing number of daytraders found today. Discovering if, and under what circumstances, the presence of daytraders would contribute to market bubbles or crashes is the type of qualitative question best suited for the computational approach. While difficult to model in agent based simulation, such questions are impossible to pose theoretically and occur too infrequently to pose empirically, thus the computational approach remains a powerful tool.

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